The pionic width of the $\omega(782)$ meson within a well-defined, unitary quantum field theory of (anti-)particles and (anti-)holes

Frieder Kleefeld[a *], **Eef van Beveren**[b †] **and George Rupp**[a ‡]

[a] *Centro de Física das Interacções Fundamentais (CFIF), Instituto Superior Técnico (IST), Edifício Ciência, Av. Rovisco Pais, P-1049-001 Lisboa, Portugal*

[b] *Departamento de Física, Universidade de Coimbra, P-3004-516 Coimbra, Portugal*

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Abstract

We investigate the indirect generation of the partial width of the $\omega(782)$ meson with respect to the reaction $\omega(782) \to \rho(770)\,\pi \to \pi^+\pi^-\pi^0$ within the context of a "Unitary Effective Resonance Model" (UERM), which has previously been applied to resonant fermionic field operators, here extended to scalar, pseudoscalar and vector bosonic fields. The $\omega(782)$ meson is described by a (quasi-)real field, while the intermediate $\rho(770)$ meson is considered to be a resonant degree of freedom, which can be treated consistently within the UERM.

In the limit of an infinitesimal width (imaginary part of self-energy), the UERM yields a consistent treatment of relativistic quantum field theory (QFT). Some aspects of the UERM of (anti-)particles and (anti-)holes are discussed.

Key words: Meson decays (ω width, ρ width, cascade process), Field theory (unitary effective resonance model, anti-particles, holes)

* kleefeld@cfif.ist.utl.pt (corresponding author)
† eef@teor.fis.uc.pt
‡ george@ajax.ist.utl.pt

1 Introduction

Pions are fundamental particles of effective formulations for the description of strong interactions, not only because they are the first discovered mediators of nuclear forces, but also since most hadrons created in experiment may decay either directly, or through a cascade of decay processes, into pions, which subsequently evaporate into photons, neutrinos, and leptons. In this article we will study the decay process of the $\omega(782)$ meson.

In the unitarised meson model (NUMM) of Ref. [1] it was found that, among the various two-meson channels which couple to the non-strange quark+anti-quark system with the $\omega(782)$ quantum numbers, the $\rho(770)\pi$ channel is the main responsible for the correct central mass position of the $\omega(782)$ meson. This model observation does not stand alone, but was already suggested as early as in 1962 by M. Gell-Mann, D. Sharp, and W.G. Wagner [2], and has been studied in a number of publications [3, 4] since. It is moreover supported by experiment, as we will explain below.

In the NUMM one considers implicit mixing of the non-strange and strange $q\bar{q}$ sectors through their common $K\bar{K}$, $K^*\bar{K}$, $K\bar{K}^*$, and $K^*\bar{K}^*$ channels. The resulting system simultaneously describes both the $\omega(782)$ and $\phi(1020)$ mesonic states, and their radial excitations. As a consequence, the NUMM $\phi(1020)$ mesonic state contains a small non-strange $q\bar{q}$ component, which fully mimics the three-pion decay width through its $\rho(770)\pi$ partial decay width. From experiment one deduces that at least 80 percent of the three-pion decay mode of the $\phi(1020)$ meson stems from a cascade process through $\rho(770)\pi$ [5, 6]. Hence, when in the NUMM the non-strange component of the $\phi(1020)$ meson cascades dominantly through $\rho(770)\pi$ into three pions, then one may safely assume that the non-strange component of its partner, the $\omega(782)$ meson, also does.

However, with ρ and π masses of about 760 MeV and 140 MeV, respectively, in the NUMM the $\rho(770)\pi$ channel is closed for the $\omega(782)$ meson. Consequently, in the NUMM one ends up with a bound state (or infinitely sharp resonance) for the model's ω meson. There are two ways out of that situation: either one assumes the existence of many ρ mesons, for instance with a Breit-Wigner-like probability distribution around the central resonance position [7], or one takes a complex value for the ρ mass. Both cases evidently describe the cascade process

$$\omega \longrightarrow \rho\pi \longrightarrow (2\pi)\pi \quad . \tag{1}$$

Substitution in the NUMM of the $\rho(770)\pi$ channel by a variety of $\rho\pi$ channels, each with a different, but real, ρ-meson mass, obviously leads to the numerical difficulty of needing to handle very many coupled channels and, moreover, to the rather ad hoc choice of their probability distribution, whereas opting for a complex ρ-meson mass gives rise to non-unitarity.

The reason for the present study is that the ω width is a clean application and test of a recently developed technique which allows for complex masses without violating unitarity. This formalism, the Unitary Effective Resonance Model (UERM), has been developed for fermionic degrees of freedom in order to consistently treat the dynamics of baryonic resonances [8].

In this paper the UERM will be extended to bosonic systems. The resulting model will then be applied to the description of the effective two-pion system with the ρ-meson quantum numbers.

The organisation of this paper is as follows. In section 2 the concept of the UERM is extended to a well-defined quantum field theory (QFT) of scalar, pseudoscalar and vector bosonic (anti-) particles and (anti-)holes with infinitesimal or non-zero width. Several interpretational points of this QFT are addressed. In section 3 we calculate the UERM result for the cascade process (1), which consistently demonstrates the indirect generation of the partial decay width of a "real" ω meson into three pions via the formation of an intermediate resonant ρ meson. The conclusions and a discussion are presented in section 4.

2 The UERM extension to bosons

The present theoretical understanding of particle dynamics at low and intermediate energies is formulated in terms of effective field theories (EFTs), i.e., QFTs of *effective* degrees of freedom, which are assumed to be the low-energy limit of underlying field theories describing the dynamics of *elementary* (point-like) degrees of freedom. Some of the effective degrees of freedom may be considered stable, like pions in strong interactions, others are experimentally known to have finite lifetimes, like the ρ meson, which shows they are complex dynamical systems under time evolution. In this paper we address the question of how to incorporate the latter effective degrees of freedom in a consistent way in the EFT for mesons. For effective fermionic degrees of freedom with finite lifetimes, like baryonic resonances, the formalism has been extensively described in Ref. [8]. Here we study the extension to effective bosonic degrees of freedom, leading to a well-defined QFT of particles, anti-particles, holes, and anti-holes.

The UERM is not only well-defined with respect to boundary conditions and unitarity, it reproduces the results obtained within the commonly used QFTs in the fermionic sector exactly, while yielding small, yet non-trivial corrections in the bosonic sector stemming from the negative-energy states of bosons here defined to be bosonic holes and anti-holes. Moreover, it puts the field-theoretic treatment of bosonic degrees of freedom on the same footing as that of the fermions, it provides a new interpretation of the meaning of anti-particles and holes, and it allows a field-theoretic treatment of effective degrees of freedom with a non-zero width. The concept of a Dirac sea is no longer necessary and the Klein-Gordon equation is allowed to have negative-energy solutions without immediate Bose-Einstein condensation of bosonic states to infinite negative energy.

Furthermore, there is a close correspondence between Fock state vectors in UERM and (anti-) Gamow states in quantum mechanics (QM). Here, the Fock state vectors of (anti-)particles in UERM correspond to Gamow states in QM, whereas the Fock state vectors of (anti-)holes in UERM correspond to anti-Gamow states in QM. As a consequence several properties of states with non-zero or infinitesimal width can also be observed and studied in quantum-mechanical models treating resonances on the basis of (anti-)Gamow states [9, 10]. The presence of anti-Gamow states in scattering theory is essential in order to consistently handle time-reversal invariance and to restore unitarity, like the existence of (anti-)holes in UERM, which can be studied in more detail using separable potentials [9].

2.1 (Anti-)particles and (anti-)holes

2.1.1 Introduction

Consider the generating functional of a real free bosonic field $\phi(x)$ with the free action $S_0[\phi]$ in Minkowski space (metric tensor $g_{\mu\nu} = \text{diag}(+,-,-,-)$):

$$\begin{aligned} Z_0 &= \int D[\phi]\, \exp\Big(i\, S_0[\phi]\Big) \\ &= \int D[\phi]\, \exp\Big(i \int d^4x\, \frac{1}{2}\Big((\partial_\mu\, \phi(x))(\partial^\mu\, \phi(x)) - m^2\, \phi^2(x)\Big)\Big)\ . \end{aligned} \tag{2}$$

This generating functional is only a well-defined convergent Gaussian path integral, if the (real) mass m of the boson is analytically continued into the complex plane by introducing an infinitesimal negative imaginary part, to yield an exponential fall off of the integrand. Hence, one has to perform the replacement $m \to M = m - i\,\varepsilon$ with ε being real, positive, and infinitesimally small.

A consequence of this substitution becomes more evident if we allow for a finite width $\Gamma/2$, i.e., $M = m - i\,\Gamma/2$. The action (and therefore also the effective action) and the Hamilton operator are no longer hermitian, which yields a non-unitary S matrix or Dyson operator. The eigenstates of the Hamilton operator (or the Fock states) are no longer orthogonal, and, simply speaking, the "bras" ($< \ldots |$) are no longer hermitian conjugates of the "kets" ($| \ldots >$). Of course, the same arguments also apply to a free fermionic field. The, at least infinitesimal, non-hermiticity of the Hamilton operator is indicating that the Fock space in QFT, or the Hilbert space in QM, is incomplete. This problem has been solved for charged fermionic resonance fields of finite width within the UERM [8] by consistently introducing additional fields in the Lagrangian in order to restore the unitarity of the theory.

In the same way as in the fermionic case, the real bosonic field $\phi(x)$ has to be replaced by a "quasi-real" field $\phi(x)$ and its hermitian conjugate $\phi^+(x)$. In order to include isospin at this stage, i.e., charged fields, we start with N "quasi-real" fields $\phi_r(x)$ $(r = 1, \ldots, N)$ of equal complex mass $M = m - i\,\Gamma/2$, yielding the following free Lagrangian for what we traditionally call N "real" (i.e. uncharged) bosons:

$$\begin{aligned} L^0_\phi(x) &= \sum_r \frac{1}{2}\Big((\partial_\mu\, \phi_r(x))(\partial^\mu\, \phi_r(x)) - M^2\, \phi_r(x)\, \phi_r(x)\Big) + \\ &+ \sum_r \frac{1}{2}\Big((\partial_\mu\, \phi^+_r(x))(\partial^\mu\, \phi^+_r(x)) - M^{*\,2}\, \phi^+_r(x)\, \phi^+_r(x)\Big)\ . \end{aligned} \tag{3}$$

The term in the first line of formula (3) is well known. It is reponsible for the description of particles and anti-particles, while the term in the second line is absent in traditional QFTs. This term is describing what we will call holes and anti-holes. As we will see later on, *with respect to movement in the forward time direction*, the real part of the energy of *particles and anti-particles* is positive, whereas the real part of the energy of *holes and anti-holes* is negative. On the other hand, *with respect to movement in the backward time direction*, the real part of the energy of *particles and anti-particles* is negative, whereas the real part of the energy of *holes and anti-holes* is positve. So *with respect to movement in the forward time direction*, particles and anti-particles *minimise* their energy, whereas holes and anti-holes *maximise* their energy. *With respect to movement in the backward time direction*, particles and anti-particles *maximise* their energy, whereas holes and anti-holes *minimise* their energy. In our language, anti-particles are objects with opposite additive quantum numbers (e.g. charge, parity, ...) as compared to the respective particles, except for the energy. Anti-holes have opposite additive quantum numbers as compared to the respective holes, except for the energy. This is why, with respect to the forward time direction, the energy of annihilation radiation of a particle and an anti-particle is positive and larger than or equal to twice the mass of the particle, whereas the energy of the annihilation radiation of a hole and an anti-hole is less than or equal to twice the (negative) mass of the hole. So, in the UERM there no longer exists an identification of anti-particles and holes.

The distinction between anti-particles and holes (or particles and anti-holes) seems to be quite arbitrary for objects with inifinitesimal width, yet for the field-theoretic description of systems of non-zero width it is crucial.

2.1.2 Canonical quantisation of "quasi-real" boson fields

The classical equations of motion are obtained in the standard manner by varying the action $S = \int d^4x\, L^0_\phi(x)$ with respect to the fields $\phi_r(x)$ and $\phi^+_r(x)$ $(r = 1, \ldots, N)$:

$$(\,\partial^2 + M^2)\,\phi_r(x) \;=\; 0 \quad , \quad (\,\partial^2 + M^{*\,2})\,\phi^+_r(x) \;=\; 0\,. \tag{4}$$

Obviously, equations (4) are pairwise complex conjugate to each other, i.e., pairwise equivalent, while each equation contains two coupled equations for the real and imaginary part of $\phi_r(x)$ and $\phi^+_r(x)$, respectively. The standard canonical conjugate momenta to the fields $\phi_r(x)$ and $\phi^+_r(x)$ are $(r = 1, \ldots, N)$

$$\Pi_r(x) = \frac{\delta\, L^0_\phi(x)}{\delta\,(\partial_0\, \phi_r(x))} = \partial_0\, \phi_r(x)\;,\;\; \Pi^+_r(x) = \frac{\delta\, L^0_\phi(x)}{\delta\,(\partial_0\, \phi^+_r(x))} = \partial_0\, \phi^+_r(x)\;. \tag{5}$$

Canonical quantisation in configuration space yields the *non-vanishing* equal-time commutation relations $(r, s = 1, \ldots, N)$

$$\begin{aligned} [\,\phi_r(\vec{x}, t), \Pi_s(\vec{y}, t)\,] &= i\,\delta^3(\vec{x} - \vec{y}\,)\,\delta_{rs} \\ [\,\phi^+_r(\vec{x}, t), \Pi^+_s(\vec{y}, t)\,] &= i\,\delta^3(\vec{x} - \vec{y}\,)\,\delta_{rs}\,. \end{aligned} \tag{6}$$

The classical equations of motion (4) can be solved by a standard Laplace transform. The corresponding transformation for the field operators reads ($\omega\,(|\vec{k}\,|) = \sqrt{|\vec{k}\,|^2 + M^2}$ with $\mathrm{Re}\,(\omega\,(|\vec{k}\,|)) \geq 0$, $k^\mu = (\omega\,(|\vec{k}\,|),\, \vec{k}\;)$, and $k^{*\,\mu} = (\omega^*(|\vec{k}\,|),\, \vec{k}\;)$)$(r = 1, \ldots, N)$

$$\begin{aligned} \phi_r(x) &= \int\!\frac{d^3k}{(2\pi)^3\; 2\,\omega\,(|\vec{k}\,|)}\;\left[\,a\,(\vec{k}, r)\; e^{-\,ikx} \;+\; c^+\,(\vec{k}, r)\; e^{ikx}\,\right] \\ \phi^+_r(x) &= \int\!\frac{d^3k}{(2\pi)^3\; 2\,\omega^*(|\vec{k}\,|)}\;\left[\,c\,(\vec{k}, r)\; e^{-\,ik^*x} \;+\; a^+\,(\vec{k}, r)\; e^{ik^*x}\,\right]. \end{aligned} \tag{7}$$

The consistent *non-vanishing* commutation relations for creation and annihilation operators in momentum space are $(r, s = 1, \ldots, N)$

$$\begin{aligned} [\,a\,(\vec{k}, r)\;,\; c^+(\vec{k}\,', s)\,] &= (2\pi)^3\, 2\,\omega\,(|\vec{k}\,|)\;\delta^3(\vec{k} - \vec{k}\,'\,)\;\delta_{rs} \\ [\,c\,(\vec{k}, r)\;,\; a^+(\vec{k}\,', s)\,] &= (2\pi)^3\, 2\,\omega^*(|\vec{k}\,|)\;\delta^3(\vec{k} - \vec{k}\,'\,)\;\delta_{rs}\,. \end{aligned} \tag{8}$$

At this point it can been seen that creation and annihilation operators are not hermitian conjugate to one another.

Before making further interpretations of creation and annihilation operators, it is useful to construct the free Hamilton operator in the standard manner, i.e.,

$$
\begin{aligned}
H^0_\phi &= \int d^3x \left[\sum_r \Big(\Pi_r(x)\ (\partial_0\, \phi_r(x)) + (\partial_0\, \phi^+_r(x))\ \Pi^+_r(x) \Big) - L^0_\phi(x) \right] \\
&= \sum_r \int d^3k\ \frac{1}{2}\, \omega\,(|\vec{k}\,|)\ \Big(c^+(\vec{k},r)\ a(\vec{k},r) + a\,(\vec{k},r)\ c^+(\vec{k},r)\ \Big) + \\
&+ \sum_r \int d^3k\ \frac{1}{2}\, \omega^*(|\vec{k}\,|)\ \Big(a^+(\vec{k},r)\ c\,(\vec{k},r) + c\,(\vec{k},r)\ a^+(\vec{k},r)\ \Big)\ .
\end{aligned}
\tag{9}
$$

Keeping in mind that the the words "creation" and "annihilation" are associated with the forward time direction, we can read off from formula (9) the simple identifications

$$
\begin{array}{lcl}
c^+ & \leftrightarrow & \text{creation operator of (uncharged) bosonic particle,} \\
a^+ & \leftrightarrow & \text{creation operator of (uncharged) bosonic hole,} \\
c & \leftrightarrow & \text{annihilation operator of (uncharged) bosonic hole,} \\
a & \leftrightarrow & \text{annihilation operator of (uncharged) bosonic particle .}
\end{array}
\tag{10}
$$

As the (anti-)particle and (anti-)hole subsectors of the Hamilton operator are not hermitian, the left and right eigenvectors of these parts of the Hamiltonian are not just related by hermitian conjugation. Hence, the Hamilton operator H^0_ϕ is "diagonal" in a more generalised sense. For example, the "diagonal" matrix elements of H^0_ϕ corresponding to one-(anti-)particle or one-(anti-) hole states, are $<0|\, a\, H^0_\phi\, c^+|0>$ and $<0|\, c\, H^0_\phi\, a^+|0>$, while the vanishing "off-diagonal" matrix elements are $<0|\, c\, H^0_\phi\, c^+|0>$ and $<0|\, a\, H^0_\phi\, a^+|0>$. A similiar discussion applies in general to the calculation of expectation values.

It is straightforward to evaluate the following identity for the Feynman propagator of an uncharged bosonic field ($r, s = 1, \ldots, N$):

$$
i\,\Delta_F(x-y)\ \ \delta_{rs} \ = \ <0|\, T\,(\, \phi_r\,(x)\, \phi_s\,(y)\,)\, |0> \, = \, i \int \frac{d^4k}{(2\,\pi)^4}\ \frac{e^{-\,i\,k(x-y)}}{k^2 - M^2}\ \ \delta_{rs}\ .
\tag{11}
$$

2.1.3 Canonical quantisation of the "complex" boson field

In order to proceed in the standard manner, we also consider the simplest case of a charged bosonic field, i.e., $N = 2$ (an example would be the $\rho^\pm$ system). We can define as usual the eigenstates of positive and negative charge by $\phi_\pm(x) = (\phi_1(x) \pm i\, \phi_2(x))/\sqrt{2}$. Inserting these states into (3) we obtain the free Lagrangian

$$
\begin{aligned}
L^0_\phi(x) &= (\partial_\mu\, \phi_+(x))(\partial^\mu\, \phi_-(x)) - M^2\ \phi_+(x)\, \phi_-(x)\ + \\
&+\ (\partial_\mu\, \phi^+_-(x))(\partial^\mu\, \phi^+_+(x)) - M^{*\,2}\ \phi^+_-(x)\, \phi^+_+(x)\ .
\end{aligned}
\tag{12}
$$

Without loss of generality we have chosen an asymmetric field ordering, which is very suitable for the quantisation of the system. If we make the replacement $\phi_-(x) \to \phi_R(x)$ and $\phi_+(x) \to \phi^+_L(x)$, we are back to the notation of Ref. [8], but now for the charged bosonic case. For the free Lagrangian we then obtain

$$\begin{aligned} L_\phi^0(x) &= (\partial_\mu \phi_L^+(x))(\partial^\mu \phi_R(x)) - M^2 \phi_L^+(x) \phi_R(x) + \\ &+ (\partial_\mu \phi_R^+(x))(\partial^\mu \phi_L(x)) - M^{*2} \phi_R^+(x) \phi_L(x) . \end{aligned} \tag{13}$$

The Lagrange equations of motion are now

$$(\partial^2 + M^2) \phi_\pm(x) = 0 \quad , \quad (\partial^2 + M^{*2}) \phi_\mp^+(x) = 0 . \tag{14}$$

The standard canonical momenta conjugate to the fields $\phi_\pm(x)$ and $\phi_\pm^+(x)$ are

$$\Pi_\pm(x) = \frac{\delta L_\phi^0(x)}{\delta(\partial_0 \phi_\pm(x))} = \partial_0 \phi_\mp(x) , \; \Pi_\pm^+(x) = \frac{\delta L_\phi^0(x)}{\delta(\partial_0 \phi_\pm^+(x))} = \partial_0 \phi_\mp^+(x) . \tag{15}$$

Canonical quantisation in configuration space yields the *non-vanishing* equal-time commutation relations

$$\begin{aligned} [\phi_\pm(\vec{x},t), \Pi_\pm(\vec{y},t)] &= i\,\delta^3(\vec{x}-\vec{y}) \\ [\phi_\pm^+(\vec{x},t), \Pi_\pm^+(\vec{y},t)] &= i\,\delta^3(\vec{x}-\vec{y}) . \end{aligned} \tag{16}$$

The Laplace transformation for the charged field operators gives

$$\begin{aligned} \phi_\pm(x) &= \int \frac{d^3k}{(2\pi)^3\, 2\,\omega(|\vec{k}\,|)} \left[a_\pm(\vec{k}\,)\, e^{-ikx} + c_\mp^+(\vec{k}\,)\, e^{ikx} \right] \\ \phi_\pm^+(x) &= \int \frac{d^3k}{(2\pi)^3\, 2\,\omega^*(|\vec{k}\,|)} \left[c_\mp(\vec{k}\,)\, e^{-ik^*x} + a_\pm^+(\vec{k}\,)\, e^{ik^*x} \right] , \end{aligned} \tag{17}$$

with the following *non-vanishing* commutation relations for creation and annihilation operators in momentum space:

$$\begin{aligned} [a_\pm(\vec{k}\,)\,,\, c_\pm^+(\vec{k}\,'\,)] &= (2\pi)^3\, 2\,\omega(|\vec{k}\,|)\, \delta^3(\vec{k}-\vec{k}\,'\,) \\ [c_\pm(\vec{k}\,)\,,\, a_\pm^+(\vec{k}\,'\,)] &= (2\pi)^3\, 2\,\omega^*(|\vec{k}\,|)\, \delta^3(\vec{k}-\vec{k}\,'\,) . \end{aligned} \tag{18}$$

The Hamilton operator is

$$\begin{aligned} H_\phi^0 &= \int d^3k\, \frac{1}{2}\, \omega(|\vec{k}\,|) \Big(c_+^+(\vec{k}\,)\, a_+(\vec{k}\,) + c_-^+(\vec{k}\,)\, a_-(\vec{k}\,) \\ &\qquad\qquad + a_+(\vec{k}\,)\, c_+^+(\vec{k}\,) + a_-(\vec{k}\,)\, c_-^+(\vec{k}\,) \Big) \\ &+ \int d^3k\, \frac{1}{2}\, \omega^*(|\vec{k}\,|) \Big(a_-^+(\vec{k}\,)\, c_-(\vec{k}\,) + a_+^+(\vec{k}\,)\, c_+(\vec{k}\,) \\ &\qquad\qquad + c_-(\vec{k}\,)\, a_-^+(\vec{k}\,) + c_+(\vec{k}\,)\, a_+^+(\vec{k}\,) \Big) . \end{aligned} \tag{19}$$

Again keeping in mind that the words “creation” and “annihilation” are associated with the forward time direction, we can read off the following identifications (starting with a positively charged particle):

$$
\begin{array}{lcl}
c_+^+ & \leftrightarrow & \text{creation operator of (charged) bosonic particle,} \\
c_-^+ & \leftrightarrow & \text{creation operator of (charged) bosonic anti-particle,} \\
a_+^+ & \leftrightarrow & \text{creation operator of (charged) bosonic anti-hole,} \\
a_-^+ & \leftrightarrow & \text{creation operator of (charged) bosonic hole,} \\
a_+ & \leftrightarrow & \text{annihilation operator of (charged) bosonic particle,} \\
a_- & \leftrightarrow & \text{annihilation operator of (charged) bosonic anti-particle,} \\
c_+ & \leftrightarrow & \text{annihilation operator of (charged) bosonic anti-hole,} \\
c_- & \leftrightarrow & \text{annihilation operator of (charged) bosonic hole .}
\end{array} \tag{20}
$$

Obviously, we can make the identifications $a_\pm = (a_1 \pm i\, a_2)/\sqrt{2}$ and $c_\pm = (c_1 \pm i\, c_2)/\sqrt{2}$. The Feynman propagator of a charged bosonic field reads

$$
i\,\Delta_F(x-y) \; = < 0|\, T\,(\,\phi_-(x)\,\phi_+(y)\,)\,|0> = \; i \int \frac{d^4k}{(2\,\pi)^4}\; \frac{e^{-\,i\,k(x-y)}}{k^2 - M^2}\;. \tag{21}
$$

2.1.4 Introduction of resonant bosonic vector fields

In Ref. [8] it was observed that in the fermionic case a consistent redefinition of spinors is necessary. Hence, in order to introduce resonant bosonic vector fields we must similarly find a consistent formulation of polarisation vectors for resonant vector mesons. The problematic point about the definition of a polarisation operator for a resonant field becomes more obvious here than in the case of a spinor. The reason is simple: one defines a momentum-dependent polarisation vector of a vector particle (quasi-real mass $M = m - i\,\varepsilon$) by boosting the constant rest-frame polarisation vector, i.e., one defines the polarisation vectors by

$$
\begin{aligned}
\varepsilon^{\,\mu\,\lambda_z}(\vec{p}\,) \;&=\; (\varepsilon^{\,0\,\lambda_z}(\vec{p}\,)\,,\,\vec{\varepsilon}^{\,\lambda_z}(\vec{p}\,)) \;=\; \Lambda^\mu_{\;\nu}(\vec{p}\,)\,\varepsilon^{\,\nu\,\lambda_z}(\vec{0}\,) \\
&=\; \left(\frac{\vec{p}\cdot\vec{\varepsilon}^{\,\lambda_z}}{M}\,,\, \vec{\varepsilon}^{\,\lambda_z} + \frac{\vec{p}\cdot\vec{\varepsilon}^{\,\lambda_z}}{M\,(\omega(|\vec{p}\,|)+M)}\,\vec{p}\,\right)\;.
\end{aligned} \tag{22}
$$

Here, μ is a Lorentz index with $\mu = 0,1,2,3$ and $\lambda_z = 0, \pm 1$ is the polarisation index. The properties of the three polarisation vectors of such a massive vector boson are well known:

$$
\begin{aligned}
p_{\,\mu}\Big|_{p^0=\omega(|\vec{p}\,|)}\; \varepsilon^{\,\mu\,\lambda_z}(\vec{p}\,) \;&=\; 0 \\
(-1)^{\lambda_z}\,\varepsilon^{\,-\,\lambda_z}(\vec{p}\,)\cdot\varepsilon^{\,\lambda_z'}(\vec{p}\,) \;&=\; (-1)^{\lambda_z}\,\varepsilon^{\,\mu\,-\,\lambda_z}(\vec{p}\,)\,\varepsilon_{\,\mu}^{\;\;\lambda_z'}(\vec{p}\,) \;=\; -\,\delta^{\,\lambda_z\lambda_z'} \\
\sum_{\lambda_z}\,(-1)^{\lambda_z}\,\varepsilon^{\,\mu\,\lambda_z}(\vec{p}\,)\,\varepsilon^{\,\nu\,-\,\lambda_z}(\vec{p}\,) \;&=\; -\,g^{\,\mu\nu} + \frac{p^{\,\mu}p^{\,\nu}}{M^{\,2}}\;.
\end{aligned} \tag{23}
$$

The question is now how to extend this description to a resonant particle with complex mass $M = m - i\,\Gamma/2$. Or in other words: what is the correct boost matrix for a resonant field? Is it the real boost matrix based on velocities $\vec{\beta}$ or is it the complex boost matrix based on momenta, energies, and complex masses? As the definition of the orbital angular momentum is based on momenta ($\vec{L} = \vec{r} \times \vec{p}$) and not on velocities, we prefer here to choose the complex boost matrix based on momenta, energies, and complex masses, i.e., we make the definition

$$\Lambda^{\mu}_{\ \nu}(\vec{p}) \;=\; \begin{pmatrix} \dfrac{\omega(|\vec{p}\,|)}{M} & \dfrac{\vec{p}^{\,T}}{M} \\ \\ \dfrac{\vec{p}}{M} & 1_3 \;+\; \dfrac{\vec{p}\;\vec{p}^{\,T}}{M\,(M+\omega(|\vec{p}\,|))} \end{pmatrix} . \tag{24}$$

The consequence is, of course, that a boost of a resonance field from a real space-time point will lead to complex space-time, which is not so unnatural though, as a resonant particle will decay in the future, i.e., it will disappear from space-time. Of course, it would be interesting to discuss what is the meaning of the words energy conservation (or, in general, the conservation of quantum numbers), simultaneity, and causality for resonant fields, and to find the answer to the question where in the complex space-time is the rest frame of a moving resonant particle like the ρ meson.

Returning to the introduction of bosonic field operators, we can now consistently write down the Laplace transformation of bosonic resonant vector field operators like the ρ meson (χ_{t_z} represent the isospinors):

$$\begin{aligned} \phi^{\mu}(x) \;=\; \sum_{t_z}\sum_{\lambda_z} & \int\frac{d^3k}{(2\pi)^3\;2\,\omega\,(|\vec{k}\,|)} \\ & \Big[\,\varepsilon^{\mu\lambda_z}(\vec{k}\,)\;\chi_{t_z}\;a\,(\vec{k},\lambda_z,t_z)\;e^{-\,ikx} \\ & +(-1)^{\lambda_z}\,\varepsilon^{\mu\,-\lambda_z}(\vec{k}\,)\;\chi^{+}_{t_z}\;c^{+}\,(\vec{k},\lambda_z,t_z)\;e^{ikx}\,\Big] \\ \phi^{+\mu}(x) \;=\; \sum_{t_z}\sum_{\lambda_z} & \int\frac{d^3k}{(2\pi)^3\;2\,\omega^{*}(|\vec{k}\,|)} \\ & \Big[\,(-1)^{\lambda_z}(\varepsilon^{\mu\,-\lambda_z}(\vec{k}\,))^{*}\;\chi_{t_z}\;c\,(\vec{k},\lambda_z,t_z)\;e^{-\,ik^{*}x} \\ & \qquad +(\varepsilon^{\mu\lambda_z}(\vec{k}\,))^{*}\;\chi^{+}_{t_z}\;a^{+}\,(\vec{k},\lambda_z,t_z)\;e^{ik^{*}x}\,\Big]\,. \end{aligned} \tag{25}$$

Using these field operators, it is now straightforward to calculate the corresponding Feynman propagator ($\phi^{\mu}\,(x) = \sum\limits_{t_z}\sum\limits_{\lambda_z}\phi^{\mu\,\lambda_z}_{t_z}\,(x)$):

$$\begin{aligned} i\,\Delta^{\mu\nu}_{F}(x-y)\;\;\delta^{\lambda_z\lambda'_z}\;\;\delta_{t_z t'_z} & \;=\; <0|\,T\,(\,\phi^{\mu\,\lambda_z}_{t_z}\,(x)\,\phi^{\nu\,\lambda'_z}_{t'_z}\,(y)\,)\,|0> \;= \\ & \;=\; i\int\frac{d^4k}{(2\,\pi)^4}\;\frac{e^{-\,i\,k(x-y)}}{k^2-M^2}\;\left(-\,g^{\mu\nu}+\frac{k^{\mu}k^{\nu}}{M^{\,2}}\right)\;\delta^{\lambda_z\lambda'_z}\;\;\delta_{t_z t'_z}\;. \end{aligned} \tag{26}$$

The quantisation of resonant vector fields is analogous to the quantisation of resonant scalar fields.

2.2 Interactions

Following the discussion of the free QFTs, it is now interesting to study the consistent implementation of interactions. Let us start with a quasi-real (uncharged) bosonic QFT ($N = 1$), for which we consider the interaction (ϕ^3-theory) Lagrangian

$$\begin{aligned}
L_{\phi}(x) &= L^{0}_{\phi}(x) + L^{int}_{\phi}(x) \\
L^{0}_{\phi}(x) &= \frac{1}{2}\Big((\partial_\mu \phi(x))(\partial^\mu \phi(x)) - M^2 \phi(x)\phi(x)\Big) + \\
&+ \frac{1}{2}\Big((\partial_\mu \phi^+(x))(\partial^\mu \phi^+(x)) - M^{*2} \phi^+(x)\phi^+(x)\Big) \\
L^{int}_{\phi}(x) &= -\frac{1}{3!}\, g_3\, \phi^3(x) \quad - \frac{1}{2!}\, g_{2,1}\, \phi^2(x)\, \phi^+(x) \\
&\quad -\frac{1}{3!}\, g_3^*\, (\phi^+(x))^3 - \frac{1}{2!}\, g_{2,1}^*\, \phi(x)\, (\phi^+(x))^2 \,, \qquad (27)
\end{aligned}$$

where g_3 and $g_{2,1}$ are, for the moment, arbitrary *complex* coupling constants. The interaction terms containing both ϕ and ϕ^+ fields need special consideration, as they yield direct interactions between (anti-)particles and (anti-)holes. The consequence of these terms is that e.g. a particle can lower its energy by creating an infinite number of holes, whereas a hole can increase its energy by creating an infinite number of particles. This situation is commonly called "radiation catastrophe" and has been cured in the fermionic case by the introduction of a Dirac sea, whereas in the bosonic case the Bose-Einstein condensation of particles to infinite negative energies has been avoided by forbidding the existence of the negative-energy states of the Klein-Gordon equation. This asymmetric and arbitrary treatment of the problem of fermionic and bosonic negative-energy states in QFT is quite unsatisfactory, as was pointed out e.g. in Chapter 1 of [11].

In order to avoid the "radiation catastrophe" in a way which is symmetric for fermions and bosons, the following rule has to be made up[1]:

Direct interactions between (anti-)particles and (anti-)holes aren't allowed!

As a consequence for the Lagrangian under consideration, formula (27), we have to set $g_{2,1} = g_{2,1}^* = 0$.

[1]With respect to Ref. [8], the rule is equivalent to demanding the diagonal elements of the vertex matrix to be zero.

3 The $\omega(782) \to 3\pi$ partial decay width

3.1 The standard lowest-order chiral Lagrangian of the $\omega\rho\pi$ system

The $\omega\rho\pi$ system can be described in a simple meson picture by the phenomenological "effective" Lagrangian

$$L(x) \; = \; L^{\,0}_{\omega}(x) + L^{\,0}_{\rho}(x) + L^{\,0}_{\pi}(x) + L^{\,int}_{\omega\leftrightarrow\rho\,\pi}(x) + L^{\,int}_{\rho\leftrightarrow\pi\pi}(x) + \ldots\;. \tag{28}$$

For the determination of the respective Lagrangians, we will follow Ref. [12], based on the "hidden-symmetry approach" of Ref. [13]. First we define the SU(3)-octet matrix for the pseudoscalar mesons by:

$$P \;\simeq\; \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\,\eta_8}{\sqrt{6}} \end{pmatrix} . \tag{29}$$

Here we made the reasonable approximation $\eta_3 \simeq \pi^0$, i.e., we assume that η_8 does not contain any π^0-component. The corresponding matrix for the "ideally mixed" vector-meson-octet — "ideal mixing" is suggested to good accuracy by nature (see e.g. the *Quark Model* section of Ref. [5]) — can be introduced by:

$$V \;\simeq\; \begin{pmatrix} \dfrac{\rho^0}{\sqrt{2}} + \dfrac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\dfrac{\rho^0}{\sqrt{2}} + \dfrac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} . \tag{30}$$

The lowest-order interactions between VPP and PVV systems in the "hidden-symmetry" Lagrangian is described by the terms ($\varepsilon^{\,0123} = 1$)

$$\begin{aligned} L^{\,int}_{VPP}(x) &= i\,g\;\mathrm{Tr}\Big[\,V_\mu\,P\,(\partial^\mu P) - V_\mu\,(\partial^\mu P)\,P\,\Big] \\ L^{\,int}_{VVP}(x) &= \frac{G_\pi}{\sqrt{2}}\,\varepsilon^{\,\mu\nu\alpha\beta}\;\mathrm{Tr}\Big[\,(\partial_\mu\,V_\nu)\,(\partial_\alpha V_\beta)P\,\Big]\,. \end{aligned} \tag{31}$$

For the interaction in the $\omega\rho\pi$ system, these Lagrangians yield for "ideal mixing" of the neutral vector mesons involved

$$\begin{aligned} L^{\,int}_{\rho\pi\pi}(x) &= -\sqrt{2}\,g\;\vec{\rho}_\mu(x)\,\cdot\,\Big[\,\vec{\pi}\,(x)\times(\partial^\mu\vec{\pi}\,(x))\,\Big] \\ L^{\,int}_{\omega\rho\pi}(x) &\simeq G_\pi\,\varepsilon^{\,\mu\nu\alpha\beta}\,(\partial_\mu\,\omega_\nu(x))\,(\partial_\alpha\vec{\rho}_\beta\,(x))\cdot\vec{\pi}\,(x)\;, \end{aligned} \tag{32}$$

Henceforth we will use for couplings the notation of Ref. [12], i.e., $g = g_{\rho\pi\pi}/\sqrt{2}$, $f = \sqrt{2}\,F_\pi \simeq 132$ MeV with $F_\pi \simeq 93$ MeV and $G_\pi = \sqrt{2}\,g\,g_{\omega\pi^0\gamma}$. The order of accuracy of the relation between the couplings and the vector-meson masses of the "hidden-symmetry" Lagrangian, i.e., $m_\rho^2 \simeq m_\omega^2 \simeq m_V^2 = 2\,g^2 f^2 \simeq 0.60$ GeV^2, suggests to what accuracy our later results have to be interpreted.

Lagrangian (28) is quasi-hermitian and consists of quasi-real meson fields only. If one tries to take into account the observed finite decay width Γ_ρ of the ρ meson into two pions, one will have to include higher-loop terms upto infinite order in perturbation theory to simulate the resonant ρ intermediate state.

3.2 Determination of $\Gamma(\omega \to \rho\pi \to \pi\pi\pi)$ from an effective Lagrangian based on the well-defined QFT

In order to perform a tree-level calculation without loops[2], which is containing the essential information of the calculation within a quasi-real theory to all orders of perturbation theory, one can replace the Lagrangian of quasi-real fields by an effective Lagrangian containing resonant fields, which then have to be treated consistently within the well-defined QFT.

In practical calculations, one may obtain such a physically realistic ρ resonance in a coupled-channel approach, by also including several virtual two-meson channels, as for instance is done in Ref. [1]. This ρ resonance will then serve as an input in the effective-Lagrangian approach to calculate the leading-order ω decay width.

The ρ meson is described within such an effective Lagrangian by a resonant degree of freedom with a complex constant[3] "mass" $M_\rho = m_\rho - i\Gamma_\rho/2$, whereas the ω and π mesons, due to their comparatively small decay widths, can be treated as quasi-real fields. The effective Lagrangian of the $\omega\rho\pi$ system in the well-defined QFT will then be

$$
\begin{aligned}
L'(x) \;=\; & \Big(-\frac{1}{4}\,\omega_{\mu\nu}(x)\,\omega^{\mu\nu}(x) + \frac{1}{2}\,(m_\omega - i\,\varepsilon)^2\;\omega_\mu(x)\,\omega^\mu(x) \\
& - \frac{1}{4}\,\vec{\rho}_{\mu\nu}(x)\cdot\vec{\rho}^{\,\mu\nu}(x) + \frac{1}{2}\,M_\rho^2\;\vec{\rho}_\mu(x)\cdot\vec{\rho}^{\,\mu}(x) \\
& + \frac{1}{2}\Big((\partial_\mu\,\vec{\pi}\,(x))\cdot(\partial^\mu\,\vec{\pi}\,(x)) - (m_\pi - i\,\varepsilon)^2\;\vec{\pi}\,(x)\cdot\vec{\pi}\,(x)\Big) \\
& + G'_\pi\,\varepsilon^{\mu\nu\alpha\beta}\,(\partial_\mu\,\omega_\nu(x))\,(\partial_\alpha\vec{\rho}_\beta\,(x))\cdot\vec{\pi}\,(x) \\
& - \sqrt{2}\,g'\;\vec{\rho}_\mu(x)\cdot\Big[\vec{\pi}\,(x)\times(\partial^\mu\vec{\pi}\,(x))\Big] + \ldots\Big) + \text{h.c.}\,.
\end{aligned}
\tag{33}
$$

Some important questions are now: How will the parameters G'_π and g' of the effective Lagrangian change when compared to the original Lagrangian? Will they develop a complex phase? What are the compensating interaction terms in the effective Lagrangian which guarantee the quasi-reality of the effective action in the (anti-)particle and the (anti-)hole sector. How will the phase-space integral change, if there are resonant fields in the final state? Some of these questions we try to answer in the following discussion. For simplicity we choose $G'_\pi \simeq G_\pi$ and $g' \simeq g$. If the thus calculated partial ω width into three pions turns out to be within the range of the experimental measurements, then this assumption should be reasonable, otherwise it will have to be reconsidered.

Using the introduced effective Lagrangian $L'(x)$, the partial decay width $\Gamma(\omega \to \rho\pi \to \pi\pi\pi)$ is calculated to lowest, though leading, order by (see e.g. p. 80 in Ref. [15])($s = p^2 = m_\omega^2$)

$$
\begin{aligned}
\Gamma_{\omega\to\rho\pi\to\pi\pi\pi}(\vec{p}\,) \;\simeq\; & \frac{1}{2\sqrt{s}}\,\frac{1}{3!}\,\frac{1}{3}\sum_{\lambda_z}\sum_{t_{z1}}\sum_{t_{z2}}\sum_{t_{z3}} \\
& \frac{1}{(2\pi)^5}\int\frac{d^3k_1}{2\,\omega_\pi(|\vec{k}_1|)}\,\frac{d^3k_2}{2\,\omega_\pi(|\vec{k}_2|)}\,\frac{d^3k_3}{2\,\omega_\pi(|\vec{k}_3|)}\;\delta^4(k_1+k_2+k_3-p) \\
& |<\pi(\vec{k}_1,t_{z1})\,\pi(\vec{k}_2,t_{z2})\,\pi(\vec{k}_3,t_{z3})|\,T\Big[i^2 : L'^{\,int}_{\rho\pi\pi}(0) : : S'^{\,int}_{\omega\rho\pi} :\Big]\,|\,\omega(\vec{p},\lambda_z)>|^2\,.
\end{aligned}
\tag{34}
$$

[2]This is in the spirit of the loop shrinking that appears in [14] in the treatment of the LσM Lagrangian and the extraction of finite information out of its lowest-order loop diagrams.

[3]This assumption of course neglects the momentum dependence of the pole paramaters m_ρ and Γ_ρ.

To be more explicit, the in- and out-state vectors in terms of creation and annihilation operators are given by

$$
\begin{aligned}
<\pi(\vec{k}_1,t_{z1})\,\pi(\vec{k}_2,t_{z2})\,\pi(\vec{k}_3,t_{z3})| &= <0|\,a_{\pi}(\vec{k}_3,t_{z3})\;a_{\pi}(\vec{k}_2,t_{z2})\;a_{\pi}(\vec{k}_1,t_{z1}) \\
|\,\omega(\vec{p},\lambda_z)> &= c^{+}_{\omega}(\vec{p},\lambda_z)\,|0>\;. \qquad (35)
\end{aligned}
$$

The T matrix of the process can be simplified by Wick's theorem[4]:

$$
\begin{aligned}
i\,T_{fi} &= <\pi(\vec{k}_1,t_{z1})\,\pi(\vec{k}_2,t_{z2})\,\pi(\vec{k}_3,t_{z3})|T\Big[i^2 : L'^{\,int}_{\rho\pi\pi}(0) :: S'^{\,int}_{\omega\rho\pi} :\Big]|\,\omega(\vec{p},\lambda_z)> \\
&= i^2 \int d^4z\; <\pi(\vec{k}_1,t_{z1})\,\pi(\vec{k}_2,t_{z2})\,\pi(\vec{k}_3,t_{z3})| \\
&\qquad T\Big[: \sqrt{2}\,g'\;\vec{\rho}_{\bar{\mu}}(x)\;\cdot\Big[\,(\partial^{\bar{\mu}}\vec{\pi}\,(x))\times\vec{\pi}\,(x)\Big]\Big|_{x=0} : \\
&\qquad : G'_{\pi}\;\varepsilon^{\mu\nu\alpha\beta}\;(\partial_{\mu}\,\omega_{\nu}(z))\;(\partial_{\alpha}\vec{\rho}_{\beta}\,(z))\cdot\vec{\pi}\,(z) : \Big]\,|\,\omega(\vec{p},\lambda_z)> \\
&= 6\,i^2 \int d^4z\;\; e^{i\,x(k_1+k_2)+i\,z(k_3-\,p)} \\
&\qquad i\int\frac{d^4k}{(2\,\pi)^4}\;\frac{e^{-i\,k(x-z)}}{k^2-M_\rho^2}\;\left(-\,g_{\,\bar{\mu}\beta}+\frac{k_{\,\bar{\mu}}k_{\,\beta}}{M_\rho^2}\right) \\
&\qquad \sqrt{2}\,g'\;G'_{\pi}\;\varepsilon^{\mu\nu\alpha\beta}\;\varepsilon_{\,t_{z1}\,t_{z2}\,t_{z3}}\;i\,k_1^{\bar{\mu}}(-\,i)\,p_{\,\mu}\,\varepsilon^{\,\lambda_z}_{\,\nu}(\vec{p})_{\omega}\;(i\,k_{\alpha})\Big|_{x=0} \\
&= 6\int d^4k\;\int\frac{d^4z}{(2\,\pi)^4}\;\; e^{i\,z(k+k_3-\,p)}\;\frac{1}{k^2-M_\rho^2}\;\left(-\,k_{1\,\beta}+\frac{k_1\cdot k\;k_{\,\beta}}{M_\rho^2}\right) \\
&\qquad \sqrt{2}\,g'\;G'_{\pi}\;\varepsilon_{\,t_{z1}\,t_{z2}\,t_{z3}}\;\varepsilon^{\mu\nu\alpha\beta}\;\;p_{\,\mu}\,\varepsilon^{\,\lambda_z}_{\,\nu}(\vec{p})_{\omega}\;k_{\,\alpha} \\
&= 6\int d^4k\;\;\delta^4(k+k_3-\,p)\;\;\frac{1}{k^2-M_\rho^2}\;\;(-\,k_{1\,\beta}) \\
&\qquad \sqrt{2}\,g'\;G'_{\pi}\;\varepsilon_{\,t_{z1}\,t_{z2}\,t_{z3}}\;\varepsilon^{\mu\nu\alpha\beta}\;\;p_{\,\mu}\,\varepsilon^{\,\lambda_z}_{\,\nu}(\vec{p})_{\omega}\;k_{\,\alpha} \\
&= 6\,\sqrt{2}\,g'\;G'_{\pi}\;\varepsilon_{\,t_{z1}\,t_{z2}\,t_{z3}}\;\frac{\varepsilon^{\mu\nu\alpha\beta}\;\;p_{\,\mu}\,\varepsilon^{\,\lambda_z}_{\,\nu}(\vec{p})_{\omega}\;k_{3\,\alpha}\;k_{1\,\beta}}{(p-\;k_3)^2-M_\rho^2}\;. \qquad (36)
\end{aligned}
$$

Before proceeding it is useful to make a short remark: so far all partial derivatives acting on the resonant ρ fields were acting directly on the exponential function of the Laplace transform of the ρ propagator. As a consequence, the partial derivatives were replaced by i times plus or minus some real momentum. Therefore, we had no difficulties afterwards to replace the configuration integrals over these exponentials by four-momentum-conserving δ-distributions and to perform the final four-momentum integration in the variable k. *However, if we had some resonant fields*

[4]Within the developed formalism for resonant fields, the proof of the validity of Wick's theorem is straightforward, but for the (anti-)hole sector of the theory one has to replace the time-ordered particle propagators by the anti-time-ordered hole propagators.

in the initial or final states, the derivatives would act on the plane-wave exponentials of these fields, which would lead to a replacement of the derivatives by complex four-momenta. Even worse: the plane-wave exponentials would enter the configuration integrals, which would give four-momentum-conserving δ-distributions. As they would now contain complex four-momenta, the resulting integrals would no longer be simple δ-distributions, i.e., they could not be easily integrated away. The conclusion is that, for *intermediate* resonance fields we have no difficulties with the four-momentum-conserving δ-distributions, whereas for resonance fields in the *initial* or *final* state we run into problems. Even worse, the overall four-momentum conservation in the phase-space integral would have to be reformulated for resonant fields in the final state.

The next step is to perform the spin-isopspin averaging of $|i\,T_{fi}|^2$. We obtain:

$$\begin{aligned}\overline{|i\,T_{fi}|^2} &= \frac{1}{3!}\,\frac{1}{3}\sum_{\lambda_z}\sum_{t_{z1}}\sum_{t_{z2}}\sum_{t_{z3}}|i\,T_{fi}|^2 \\ &= \frac{1}{3}\sum_{\lambda_z}\Big|\,6\,\sqrt{2}\,g'\,G'_\pi\,\frac{\varepsilon^{\mu\nu\alpha\beta}\,p_\mu\,\varepsilon^{\lambda_z}_\nu(\vec{p}\,)_\omega\,k_{3\,\alpha}\,k_{1\,\beta}}{(p-\,k_3)^2-M_\rho^2}\Big|^2\,. \end{aligned}\tag{37}$$

At this point we again have to make a remark. Only because the decaying ω meson is treated as a real particle, i.e., $\Gamma_\omega=\varepsilon$, we can use the following identity for its polarisation vector:

$$\sum_{\lambda_z}\;\varepsilon^{\lambda_z}_\nu(\vec{p}\,)_\omega\;\;(\varepsilon^{\lambda_z}_{\bar\nu}(\vec{p}\,)_\omega)^*\;=\;-\,g_{\nu\,\bar\nu}+\frac{p_\nu\,p_{\bar\nu}}{m_\omega^2}\,. \tag{38}$$

As already described above, in general the relation $(\varepsilon^{\lambda_z}_\mu(\vec{p}\,))^*=(-1)^{\lambda_z}\;\varepsilon^{-\lambda_z}_\mu(\vec{p}\,)$ is no longer valid for objects with complex mass and has to be modified. Using these results we continue the spin-isospin averaging. In the end, we obtain the following expression for the partial width:

$$\begin{aligned}\Gamma_{\omega\to\rho\pi\to\pi\pi\pi}(\vec{p}\,) \simeq{}& \frac{1}{2\,\sqrt{s}}\;\frac{1}{(2\pi)^5}\int\frac{d^3k_1}{2\,\omega_\pi(|\vec{k}_1|)}\;\frac{d^3k_2}{2\,\omega_\pi(|\vec{k}_2|)}\;\frac{d^3k_3}{2\,\omega_\pi(|\vec{k}_3|)} \\ &\delta^4(k_1+k_2+k_3-p)\;\frac{1}{3}\left|\frac{6\,\sqrt{2}\,g'\,G'_\pi}{(p-\,k_3)^2-M_\rho^2}\right|^2\;\begin{vmatrix} p^2 & p\cdot k_3 & p\cdot k_1 \\ p\cdot k_3 & m_\pi^2 & k_1\cdot k_3 \\ p\cdot k_1 & k_1\cdot k_3 & m_\pi^2 \end{vmatrix}\,. \end{aligned}\tag{39}$$

In order to evaluate the phase-space integral, it is useful to introduce the Lorentz invariants $s=p^2=m_\omega^2$ and

$$s_1=(k_1+k_2)^2=(p-\,k_3)^3\quad,\quad s_2=(k_2+k_3)^2=(p-\,k_1)^2\,. \tag{40}$$

The following Lorentz-invariant scalar products of four-momenta are relevant to our calculation:

$$\begin{aligned}&2\;\;p\cdot k_1=m_\pi^2+s-\,s_2\;\;,\;\;2\,p\cdot k_3=m_\pi^2+s-s_1\;\;, \\ &2\;\;k_1\cdot k_3=m_\pi^2+s-\,s_1-s_2\,. \end{aligned}\tag{41}$$

Application of these scalar products yields the following expression for the decay width:

$$\begin{aligned}&\Gamma_{\omega\to\rho\pi\to\pi\pi\pi}(\vec{p}\,)\quad\simeq \\ &\simeq\;\frac{1}{2\,\sqrt{s}}\;\frac{1}{(2\pi)^5}\;\frac{\pi^2}{4\,s}\int_{(2\,m_\pi)^2}^{(\sqrt{s}-\,m_\pi)^2}ds_1\int_{s_2^-}^{s_2^+}ds_2\;\;\frac{1}{12}\left|\frac{6\,\sqrt{2}\,g'\,G'_\pi}{s_1-M_\rho^2}\right|^2 \\ &\Big[\,-\,s_1^2\,s_2+s_1\,s_2(s-s_2+3\,m_\pi^2)-m_\pi^2(s-\,m_\pi^2)^2\,\Big]\,, \end{aligned}\tag{42}$$

with $(\lambda(x,y,z) = x^2+y^2+z^2 - 2\,x\,y - 2\,y\,z - 2\,z\,x)$

$$s_2^{\pm} \;=\; \frac{1}{2\,s_1}\,\left[\, s_1\,(s - s_1 + 3\,m_\pi^2) \;\pm\; \sqrt{\lambda(s,m_\pi^2,s_1)\,\lambda(s_1,m_\pi^2,m_\pi^2)}\,\right]\,. \tag{43}$$

After performing the s_2 integration, we obtain $(\lambda(s_1,m_\pi^2,m_\pi^2) = s_1\,(s_1 - 4\,m_\pi^2)\,)$

$$\Gamma_{\omega\to\rho\pi\to\pi\pi\pi}(\vec{p}\,) \;\simeq\; \frac{1}{2\,\sqrt{s}}\,\frac{1}{(2\pi)^5}\,\frac{\pi^2}{4\,s} \int_{(2\,m_\pi)^2}^{(\sqrt{s}-\,m_\pi)^2} ds_1\,\left|\frac{6\,\sqrt{2}\,g'\,G'_\pi}{s_1 - M_\rho^2}\right|^2\,\frac{\sqrt{\lambda^3(s,m_\pi^2,s_1)\,\lambda^3(s_1,m_\pi^2,m_\pi^2)}}{72\,s_1^2}\,. \tag{44}$$

For the s_1 integration we have to rewrite the modulus squared of the ρ propagator in terms of the real and imaginary parts of the complex mass $M_\rho = m_\rho - i\,\Gamma_\rho/2$, i.e.,

$$\frac{1}{|\,s_1 - M_\rho^2\,|^2} \;=\; \frac{1}{|\,s_1 - (m_\rho - i\,\frac{\Gamma_\rho}{2}\,)\,|^2} \;=\; \frac{1}{(\,s_1 - m_\rho^2 + \frac{\Gamma_\rho^2}{4})^2 + m_\rho^2\,\Gamma_\rho^2}\,. \tag{45}$$

With this expression, the partial decay width is finally given by

$$\Gamma_{\omega\to\rho\pi\to\pi\pi\pi}(\vec{p}\,) \;\simeq\; \frac{1}{2\,\sqrt{s}}\,\frac{1}{(2\pi)^5}\,\frac{\pi^2}{4\,s}\,\frac{|\,6\,\sqrt{2}\,g'\,G'_\pi\,|^2}{72} \int_{(2\,m_\pi)^2}^{(\sqrt{s}-\,m_\pi)^2} ds_1\;\frac{\sqrt{\lambda^3(s,m_\pi^2,s_1)\,\lambda^3(s_1,m_\pi^2,m_\pi^2)}}{s_1^2\,(\,(\,s_1 - m_\rho^2 + \frac{\Gamma_\rho^2}{4})^2 + m_\rho^2\,\Gamma_\rho^2\,)}\,. \tag{46}$$

The integral can easily be evaluated numerically, after having set $\sqrt{s} = m_\omega$ and remembering the identities $\lambda(s,m_\pi^2,s_1) = (s-\,m_\pi^2-\,s_1)^2 - 4\,m_\pi^2\,s_1$ and $\lambda(s_1,m_\pi^2,m_\pi^2) = s_1\,(s_1 - 4\,m_\pi^2)$. Substituting $m_\omega = 0.782$ GeV, $m_\pi = 0.140$ GeV, $m_\rho = 0.770$ GeV, and $\Gamma_\rho = 0.151$ GeV, we obtain for the integral the result 6.94×10^{-3} $(\mathrm{GeV})^6$.

Furthermore, the coupling $g' \simeq g = 4.2\,(\pm0.1)$ has been determined from the decay width $\Gamma(\rho^0 \to \pi^+\pi^-) = g^2 q_\pi^3/(3\pi\,m_\rho^2) = 151.2 \pm 1.2$ MeV with $q_\pi = \sqrt{\lambda(m_{\rho^0}^2, m_{\pi^+}^2, m_{\pi^-}^2)/(4\,m_{\rho^0}^2)} = \sqrt{(m_{\rho^0}^2 - 4\,m_{\pi^\pm}^2)/4}$, while $G'_\pi \simeq G_\pi = \sqrt{2}\,g\,g_{\omega\pi^0\gamma}$ follows from the partial decay width $\Gamma(\omega \to \pi^0\gamma) \simeq \alpha_{\mathrm{QED}}\,g_{\omega\pi^0\gamma}^2\,q_\gamma^3/3 = 0.74$ MeV, where $q_\gamma = \sqrt{\lambda(m_\omega^2, m_{\pi^0}^2, 0)/(4\,m_\omega^2)} = (m_\omega^2 - m_{\pi^0}^2)/(2m_\omega)$ $= 0.379$ GeV (see also Ref. [12, 16]). With $\alpha_{\mathrm{QED}} \simeq 1/137$, we thus get

$$g_{\omega\pi^0\gamma} = \sqrt{\frac{3\Gamma(\omega\to\pi^0\gamma)}{\alpha_{\mathrm{QED}}\,q_\gamma^3}} = 2.36\ \mathrm{GeV}^{-1},$$

which gives $G'_\pi \simeq 14.0\ \mathrm{GeV}^{-1}$. Combining all, we finally obtain our prediction

$$\Gamma_{\omega\to\rho\pi\to\pi\pi\pi} \;=\; 6.34\ \mathrm{MeV}\,.$$

In order to compare this with experiment, we must first try to determine which fraction of the total ω width of 8.44 MeV [5] is due to the cascade $\omega \to \rho\pi \to \pi\pi\pi$. Now, also according to the most recent *Review of Particle Properties* [5], the ω ows about 89% of its width to three-pion decays. However, part of this may be the result of direct, *cascadeless* 3π decays, which can be understood as OZI decays with the "simultaneous" creation of *two* $q\bar{q}$ pairs, or as a "quark-box contribution" in the language of Lucio-Martinez *et al.*, Ref. [3]. Unfortunately, there is no experimental information on the proportion of "direct" vs. (virtual) "cascade" decays of the ω. Nevertheless, as already mentioned in the introduction, such information does exist for the $\phi(1020)$, according to which the $\rho\pi$ mode accounts for at least 80% of the total three-pion decays, provided that interference between the $\rho\pi$ and (direct) 3π modes is neglected [5, 6]. Considering that the 3π decays of the $\phi(1020)$ are due to its (small) non-strange ($n\bar{n}$) component, it seems reasonable to extrapolate this finding to the $\omega(782)$. Thus, we get an estimated experimental width

$$\Gamma^{\rm exp}_{\omega\to\rho\pi\to\pi\pi\pi} \geq 0.80 \times 0.89 \times 8.44 \text{ MeV} = 6.0 \text{ MeV},$$

with an "upper bound" of $0.89 \times 8.44 = 7.5$ MeV, which fully agrees with our UERM prediction.

Also note that our G' of 14.0 GeV^{-1} is in good agreement with the experimental data (14 ± 2 GeV^{-1} [4]), in particular as compared to the interpolated result for $g_{\omega\rho\pi}$ in Table I of Ref. [3] for a $\Gamma^{\rm GSW}(\omega \to 3\pi)$ equal to the value we just found.

4 Conclusion and discussion

In the present paper we have extended a formalism, previously applied to fermions, to bosonic systems, i.e., charged and uncharged scalar, pseudoscalar and vector mesons. This method allows to handle, in a field-theoretic framework, resonant degrees of freedom in the intermediate state on the same footing as stable particles.

A simple yet topical application of the method to the three-pion decay of the ω meson, via the cascade process (1), results in an ω partial decay width in full agreement with experiment.

In principle, the formalism can be applied to many other known cascade processes, not only by effective field-theoretic methods, but also in quantum-mechanical coupled-channel approaches.

As already stated above, the application of the UERM to processes with resonant particles in the initial or final state is a task for future investigation, which may have a large impact on the understanding of how to construct relativistic optical "potentials".

As an outlook, we also want to stress here several points that have non-trivial consequences for standard QFT, as induced by the UERM in the limit of vanishing width:

The extension of the UERM to massless fermions (like (anti-)neutrinos) is straightforward and well-defined, if one keeps the width of such degrees of freedom finite. Yet the so called "chiral symmetry" of such systems gets, within the UERM, a completely new meaning, as it leads to an interchange of the (anti-)particle and (anti-)hole sector of the theory, which cannot be resolved in an equivalent and adequate way by standard QFT. A simple example is the simultaneous existence of a conserved axial current (CAC) and an inequivalent partially conserved axial current (PCAC) in such a vector-current-conserving (CVC) theory.

The UERM treatment of massless bosons is only well-defined, if both the imaginary *and* real part of their self-energies stay at least infinitesimally non-zero. In case of a vanishing real part of the self-energy of bosons, the causality properties of (anti-)particles and (anti-)holes coincide, giving rise to formal pathologies, to be resolved only by careful limiting procedures.

Once this kind of complications are under control, it is a challenging task to extend the UERM to (massless) gauge bosons, and to investigate how to introduce and understand abelian and non-abelian local gauge symmetries within the UERM. After all, the consideration of gauge theories with chiral fermions within the UERM will lead to a much more well-defined and unambiguous treatment of the anomaly sector of such chiral gauge theories than in standard QFT.

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